International Journal of Mathematics Trends and Technology
ISSN: 2231 – 5373 /doi:10.14445/22315373/IJMTT-V67I6P521
Volume 67 Issue 6, 195-197, June, 2021
© 2021 Seventh Sense Research Group®

# Some Studies of spgTc space & Continuous Functions

Tippeshi.V. Marigoudar[#1], Imtiyaz. M. Teredhahalli[*2]

[#1]*Department of Mathematics, Nehru Arts,Science and Commerce College,Hubli-580020. Karnataka State, India*
[#2]*Department of Mathematics, Govt. First Grade College, Haveri -581110. Karnataka State, India*

**Abstract:** *In this paper is to introduce the Tc Spaces and studied some of their properties and characterizations.*

**Keywords –** *Topological spaces, sgp(X,T), Tc space and sgpTc space*

## I. Introduction

In 1982, Malghan [7] introduced and studied the concept of generalized closed functions. After that several topologists like Sundaram [11] , Noiri [8], Biswas [1], Mashhour et al [5,6], Noiri et al [9], Devi et al [3, 2], Gnanambal [4], Sheik John[88], Veera Kumar [12] and Rajamani and Viswanathan [10] and introduced and studied generalized open functions, semi closed functions, semi open functions, α-open functions, gp-closed functions, αg-closed and gs-closed functions, gpr-closed functions, ω-closed and ω-open functions, g*-closed functions, and αgs-closed and αgs-open functions respectively.

## II. Preliminaries

Throughout the thesis (X, T) ,(Y, Ω) and (Z,**ρ**) denote topological spaces on which no separation axioms are assumed unless explicitly stated and they simply written as X and Y respectively. All sets are considered to be subsets to topological spaces.

**Definition 2.1:** A topological space (X, T) is said to be a sgp$T_c$-space if every sgp -closed set is closed set.

**Example 2.2:** Let X = {p, q, r} and τ = {X, ϕ, {p}, {q}, {p, q}}. Then the space X is a sgp$T_c$-space.

**Theorem 2.3:** If a: X → Y and b: Y → Z are two sgp-continuous functions and Y is sgp$T_c$-space, then b0a : X → Z is sgp –continuous function.

**Proof:** Let F be a closed set in Z. Then $b^{-1}(F)$ is sgp-closed set in Y since g is sgp-continuous function. As Y is sgp$T_c$-space, $b^{-1}(F)$ is closed set in Y. Again since 'a' is sgp-continuous, $a^{-1}(b^{-1}(F)) = (boa)^{-1}(F)$ is sgp-closed in X. Hence b0a: X → Z is sgp-continuous.

**Definition 2.4**: A function a: (X, T)→ (Y, Ω) is called pre- sgp-continuous if $a^{-1}(G)$ is sgp-closed in X for every α-closed set G in Y

**Example 2.5:** Let X = Y = {p, q, r}, T = {X,ϕ, {p}, {p, r}} and Ω = {Y,ϕ, {p}}. Let f: (X, T) → (Y, Ω) be an identity map. Then 'a' is pre-sgp-continuous.

**Theorem 2.6:** If a: (X, T) $\rightarrow$ (Y, $\Omega$) and b: (Y, $\Omega$) $\rightarrow$ (Z,) are pre-sgp-continuous functions and (Y, $\Omega$) is sgp$T_c$-space. Then their composition b0a: (X, T) $\rightarrow$ (Z, $\rho$) is pre-sgp-continuous.

**Proof:** Let H be an $\alpha$-closed set in (Z, $\rho$). Then $b^{-1}$(H) is sgp-closed set in (Y, $\Omega$) as 'b' is pre-sgp-continuous. Since (Y, $\Omega$) is sgp$T_c$-space, $b^{-1}$(H) is closed set and so $\alpha$-closed in (Y, $\Omega$). Again since 'a' is pre-sgp-continuous, $a^{-1}(b^{-1}(H)) =(b0a)^{-1}(H)$ is sgp-closed in (X, T).

**Theorem 2.7:** If a: (X, T) $\rightarrow$ (Y, $\Omega$) is pre-sgp-continuous, b: (Y, $\Omega$) $\rightarrow$ (Z, $\rho$) is sgp-continuous and (Y, $\Omega$) is sgp$T_c$-space. Then their composition boa: (X, T) $\rightarrow$ (Z, $\rho$) is sgp-continuous.

**Proof:** Let H be an $\alpha$-closed set in (Z, $\rho$). Then $b^{-1}$(H) is sgp-closed set in (Y, $\Omega$) as 'b' is sgp-continuous. Since (Y, $\Omega$) is sgp$T_c$-space, $b^{-1}$(H) is closed set and so $\alpha$-closed in (Y, $\Omega$). Again since 'a' is sgp-continuous, $a^{-1}(b^{-1}(H)) =(b0a)^{-1}(H)$ is sgp-closed in (X, T).

**Definition 2.8:** A function a: X $\rightarrow$ Y is called semi generalized pre- irresolute (briefly, sgp-irresolute) function if the inverse image of every sgp-closed set in Y is sgp-closed in X.

**Theorem 2.9:** Let a: X $\rightarrow$ Y be a function. If Y is sgp$T_c$-space, then the following are equivalent.

(i) a is sgp-irresolute

(ii) a is sgp-continuous.

**Proof:** (i) $\rightarrow$(ii): Follows from the Theorem 3.2.57.

Let H be any closed set in Y. Then H is sgp-closed set in Y. As 'a' is sgp-irresolute, $a^{-1}$(H) is sgp-closed set in X. Therefore 'a' is sgp-continuous function.

(ii) $\rightarrow$ (i): Let G be an sgp-closed set in Y. Since Y is sgp$T_c$-space, G is closed in Y and by hypothesis $a^{-1}$(G) is sgp-closed set in X. Therefore a is sgp-irresolute. Hence (i) holds.

**Example 2.10:** Let X ={p, q, r}, T = {X, $\phi$, {p}, {q}, {p, q}} and $\Omega$ = {Y, $\phi$, {r}, {p, r}}. Define a function a: X $\rightarrow$ Y by a(p) = r, a (q) = p and a (r) = q. Then 'a' is not a sgp-irresolute, since for the sgp-closed set {p} in Y, $a^{-1}$({p}) = {q} is not a sgp-closed set of X. However 'a' is sgp-continuous function.

**Theorem 2.11:** If a : (X, T) $\rightarrow$ (Y, $\Omega$) is $\alpha$-irresolute and (Y, $\Omega$) is sgp$T_c$-space, then 'a'is sgp-irresolute.

**Proof:** Let H be a sgp-closed set in (Y, $\Omega$). Then H is closed and so it is $\alpha$-closed set in (Y, $\Omega$) as (Y, $\Omega$) is sgp$T_c$-space. Since 'a' is $\alpha$-irresolute, $a^{-1}$(H) is $\alpha$-closed set in (X, T). Therefore $a^{-1}$(H) is sgp-closed set in (X, T). Hence 'a' is sgp-irresolute.

**Theorem 2.12:** If a: (X, T) $\rightarrow$ (Y, $\Omega$) is sgp-irresolute and (X, T) is sgp$T_c$-space, then 'a' is $\alpha$-irresolute.

**Proof:** Let H be an $\alpha$-closed set in (Y, $\Omega$). Then H is sgp-closed set in (Y, $\Omega$). Since 'a' is sgp-irresolute, $a^{-1}$(H) is sgp-closed set in (X, T). Also since (X, Y) is sgp$T_c$-space, $a^{-1}$(H) is closed and so it is $\alpha$-closed in (X, T). Hence 'a' is $\alpha$-irresolute.

**Theorem 2.13:** Let a: X $\rightarrow$ Y be onto sgp-irresolute and closed function. If X is sgp$T_c$-space, then Y is also sgp$T_c$-space.

**Proof:** Let H be a sgp-closed set in Y. Then $a^{-1}(H)$ is sgp-closed in X as 'a' is sgp-irresolute function. Since X is sgp$T_c$-space, $a^{-1}(H)$ is closed in X. Again since 'a' is closed function, $a(a^{-1}(H)) = H$ is closed in Y. Thus Y is sgp$T_c$-space.

**Theorem 2.14:** If a is sgp-continuous and b is sgp-irresolute and Y is sgp$T_c$-space, then boa: X $\rightarrow$ Z is sgp-irresolute function.

**Proof:** Let G be a sgp-closed set in Z. Then $b^{-1}(G)$ is sgp-closed in Y as g is sgp-irresolute function. Since Y is sgp$T_c$-space, $b^{-1}(G)$ is closed in Y. Again since 'a' is sgp-continuous, $a^{-1}(b^{-1}(G))$ is sgp-closed in X. But $a^{-1}(b^{-1}(G)) = (boa)^{-1}(G)$ is sgp-closed set in X. Hence boa is sgp-irresolute function.

## REFERENCES


[1] N. Biswas, on some mappings in topological spaces, Bull. Cal. Math. Soc., 61 (1969) 127-135.

[2] R. Devi, K. Balachandran and H. Maki, Semi-generalized closed maps and generalized semi-closed maps, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 14 (1993) 41-54.

[3] R. Devi, K. Balachandran and H. Maki, Generalized $\alpha$-closed maps and $\alpha$-generalized closed maps, Indian. J. Pure appl. Math., 29(1) (1998), 37-49.

[4] Y. Gnanambal and K Balachandran, on gpr- continuous functions in topological spaces, Indian. J. Pure appl. Math., 30(6) (1999) 581-593.

[5] A. S. Mash hour, M. E. Abd El-Monsef and S. N. EL-Deeb, On pre-continuous and weak pre-continuous mappings, Proc. Math and Phys. Soc. Egypt, 53 (1982) 47-53.

[6] A. S. Mashhour, I. A. Hasanein and S. N. EL-Deeb, $\alpha$-continuous and $\alpha$-open mappings, Acta Math. Hung., 41(3-4) (1983), 213-218.

[7] S. R. Malghan, Generalized closed maps, J. Karnatak Univ. Sci., 27 (1982), 82-88.

[8] T. Noiri, A generalization of closed mappings, Atti. Acad. Nasz. Lincei Rend.Cl.Sci.Fis.Mat.Natue., 54 (1973) 412-415.

[9] T. Noiri, H. Maki and J. Umehara, Generalized pre closed functions, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 19(1998) 13-20.

[10] M. Rajamani and K.Viswanathan, On $\alpha$gs-continuous maps in topological spaces. Acta Ciencia Indica, 31(1) (2005) 293-303.

[11] M. Sheik John, A study on generalizations of closed sets and continuous maps in topological spaces and bitopological spaces, Ph.D., thesis, Bharathiar University, Coimbatore (2002).

[12] M. K. R. S. Veera Kumar, Between closed sets and g-closed sets, Mem. Fac. Sci. Kochi Univ. (Math), 21 (2000) 1-19