Scholarly Research Journal for Interdisciplinary Studies,
Online ISSN 2278-8808, SJIF 2021 = 7.380, www.srjis.com
PEER REVIEWED & REFEREED JOURNAL, MAY-JUNE, 2021, VOL- 8/65

SGP- LOCALLY CLOSED SETS IN TOPOLOGICAL SPACES (PART -II)

Tippeshi. V. Marigoudar[1] & Manjunathayya. M. Holliyavar [2]

[1] *Department of Mathematics, Research Scholar, (PPMAT0168), Rayalseema University, Kurnool-518007. (A.P), India. vaishnavimarigoudar@gmail.com*

[2] *Department of Mathematics,KLE Society's SSMS College, Athani -591304 Karnataka State,India mmholliyavar@gmail.com*

Paper Received On: 21 JUNE 2021
Peer Reviewed On: 30 JUNE 2021
Published On: 1 JULY 2021
Content Originality & Unique: 87%

Abstract

The aim of this paper is to introduce the new class of sgp- locally closed sets in topological spaces and studied some of their properties and characterizations.

Keywords – *Topological spaces, SGPLC(X,τ), SGPLC*(X,τ) SGPLC**(X,τ) LC(X,τ)*
AMS Subject Classifications: 54A05, 54A10

1 Introduction

The notion of a locally closed set in a topological space was introduced by Kuratowski and Sierpinski [8]. According to Bourbaki [5], a subset A of a topological space X is called locally closed in X if it is the intersection of an open set in X and a closed set in X. Ganster and Reilly [6] used locally closed sets to define LC- Continuity and LC-irresoluteness. Balachandran, Sundaram and Maki [3] introduced the concept of generalized locally closed sets in topological spaces and investigated some of their properties. Recently Sheik John [15] introduced the three new class of sets denoted by ω-LC(X, τ), ω-LC*(X, τ) and ω-LC**(X, τ) and each of which contains LC(X, τ). Also various authors like Gnanambal [7] and Park and Park [14] have introduced α-locally closed and semi generalized locally closed sets respectively in topological spaces.

2 Preliminaries

Throughout the thesis (X, τ) and (Y, σ) denote topological spaces on which no separation axioms are assumed unless explicitly stated and they simply written as X and Y respectively. All sets are considered to be subsets to topological spaces. The complement of A is denoted by X – A. The closure and interior of a set A are denoted by Cl(A) and int(A) respectively.

The following definitions are useful in the sequel :

DEFINITION 1.1 : A subset A of a space X is said to be

(i) Semi open [9] if $A \subset Cl(Int(A))$.

(ii) semi-closed set[4] if $Int(cl(A)) \subseteq A$.

(iii) preopen [5] if $A \subset Int(Cl(A))$

(iv) preclosed [12] if $Cl(Int(A)) \subseteq A$

(v) α - open [13] if $A \subset Int(Cl(Int A))$

(vi) α - closed [11] if $Cl(Int(Cl(A))) \subseteq A$

(vii) Semi - preopen [2] (= β - open [1]) if $A \subset Cl(Int(Cl(A)))$

(viii) a semi- pre closed set [1] if $Int(cl(Int(A))) \subset A$

The family of all semi open sess (resp. semi-pre open sets) of X will be denoted by SO(X) SPO(X).

sgp-Locally Closed Sets

In this section, we introduce sgp-locally closed sets and sgp-submaximal and study some of their properties.

Definition 1.2.1: A subset A of a topological space (X, τ) is called a semi-generalized-pre locally closed set (briefly sgplc-set) if $A = S \cap F$ where S is sgp-open and F is sgp-closed.

The class of all semi-generalized-pre locally closed sets in (X,τ) is denoted by SGPLC(X,τ).

Definition 1.2.2: A subset A of a topological space (X,τ) is said to be SGPLC*-set if there exist sgp-open set S and a closed set F of (X,τ) such that $A = S \cap F$.

Definition 1.2.3: A subset A of a topological space (X,τ) is said to be SGPLC**-set if there exist an open set S and a sgp-closed set F of (X,τ) such that $A = S \cap F$.

Theorem 1.2.4: For a subset A of (X,τ), the following are equivalent:

1) $A \in SGPLC^*(X, \tau)$
2) $A = P \cap pCl(A)$ for some sgp-open set P.
3) $pCl(A)-A$ is sgp-closed.
4) $A \cup (X-pCl(A))$ is sgp-open.

Proof: (1) ⇒ (2):- Let $A\in SGPLC^*(X,\tau)$. Then there exists a sgp-open set P and a closed set F of (X,τ) such that $A = P\cap F$. Since $A \subseteq P$ and $A \subseteq pCl(A)$. Therefore we have $A \subseteq P \cap pCl(A)$.

Conversely, since $pCl(A) \subseteq F$, $P \cap pCl(A) \subseteq P \cap F= A$. Which implies that $A = P \cap pCl(A)$.

(2) ⇒ (1):- Since P is sgp-open and pCl(A) is closed.

$P \cap pCl(A) \in SGPLC^*(X,\tau)$.Which implies that $A\in SGPLC^*(X,\tau)$.

(3) ⇒ (4) :- Let $F = pCl(A)-A$. Then F is sgp-closed by the assumption and $X – F = X \cap (X – (pCl(A) – A)) = A \cup (X-pCl(A))$. But X-F is sgp-open. This shows that $A \cup (X - pCl(A))$ is sgp-open .

(4) ⇒ (3):- Let $U = A \cup (X-pCl(A))$. Since U is sgp-open, X-U is sgp-closed. $X – U = X- (A \cup (X – pCl(A))) = pCl(A) \cap (X-A) =pC(A) – A$.

Thus $pCl(A) – A$ is sgp-closed set .

(4) ⇒ (2):- Let $P = A \cup (X – pCl(A))$ Thus P is sgp-open . We prove that $A = P \cap pCl(A)$ for some sgp-open set P. $P \cap pCl(A) = (A \cup (X-pCl(A))) \cap pCl(A) = (pCl(A) \cap A) \cup (pCl(A) \cap (X – pCl(A))) = A \cup \phi = A$. Therefore $A = P \cap pCl (A)$.

(2) ⇒ (4):- Let $A = P \cap pCl (A)$ for some sgp-open set P. Then we prove that $A \cup (X-pCl(A))$ is sgp-open. Now $A \cup (X-pCl(A)) = (P \cap pCl(A)) \cup (X – pCl(A)) = P \cap (pCl(A) \cup (X – pCl(A))) = P$. Which is sgp-open. Thus $A \cup (X – pCl(A))$ is sgp-open .

Theorem 1.2.5: If $A, B\in SGPLC (X,\tau)$, then $A \cap B \in SGPLC (X,\tau)$.

Proof: From the assumptions, there exist sgp-open sets P and Q such that $A = P \cap pCl (A)$ and $B = Q \cap pCl (B)$. Then $A \cap B = (P \cap Q) \cap (pCl (A) \cap pCl(B))$. Since $P \cap Q$ is sgp-open set and $pCl (A) \cap pCl (B)$ is closed. Therefore $A \cap B \in SGPLC (X,\tau)$.

Theorem 1.2.6: If $A \in SGPLC (X,\tau)$ and B is sgp-closed set in (X,τ), then $A \cap B \in SGPLC (X,\tau)$.

Proof: Since $A \in$ SGPLC (X,τ), there exist a sgp-open set P and a sgp-closed set Q such that $A = P \cap Q$. Now $A \cap B = (P \cap Q) \cap B = P \cap (Q \cap B)$. Since P is sgp-open and $Q \cap B$ is sgp-closed, Therefore $A \cap B \in$ SGPLC (X,τ).

Theorem 1.2.7: If $A \in$SGPLC*(X,τ) and B is sgp-open (or closed) set in (X,τ), then $A \cap B \in$ SGPLC*(X,τ).

Proof: Since $A \in$ SGPLC*(X,τ), there exist a sgp-open set P and a closed set Q such that $A = P \cap Q$. Now $A \cap B = (P \cap Q) \cap B = (P \cap B) \cap Q$. Since $P \cap B$ is sgp-open and Q is closed, it follows that $A \cap B \in$ SGPLC* (X,τ).

In this case of B being a closed set, we have $A \cap B = (P \cap Q) \cap B = P \cap (Q \cap B)$. Since P is sgp-open set and $Q \cap B$ is closed. Thus $A \cap B \in$ SGPLC* (X,τ).

Theorem 1.2.8: If $A \in$SGPLC**(X,τ) and B is sgp-closed (resp. open) set in (X,τ), then $A \cap B \in$ SGPLC**(X,τ).

Proof: Since $A \in$ SGPLC**(X,τ), there exist an open set P and a sgp-closed set Q such that $A = P \cap Q$. Now $A \cap B = (P \cap Q) \cap B = P \cap (Q \cap B)$. Since P is open and $Q \cap B$ is sgp-closed, Therefore $A \cap B \in$ SGPLC**(X,τ).

In this case of B being an open set, we have $A \cap B = (P \cap Q) \cap B = (P \cap B) \cap Q$. Since $P \cap B$ is open and Q is sgp-closed, Thus $A \cap B \in$ SGPLC**(X,τ).

Theorem 1.2.9: Let (X,τ) and (Y, σ) be topological spaces.

1) If $A \in$ SGPLC (X,τ) and $B \in$ SGPLC (Y, σ), then $A \times B \in$ SGPLC $(X \times Y, \tau \times \sigma)$
2) If $A \in$ SGPLC*(X,τ) and $B \in$ SGPLC*(Y, σ), then $A \times B \in$ SGPLC* $(X \times Y, \tau \times \sigma)$.
3) If $A \in$ SGPLC**(X,τ) and $B \in$ SGPLC** (Y, σ), then $A \times B \in$ SGPLC**$(X \times Y, \tau \times \sigma)$.

Proof: 1) Let $A \in$ SGPLC (X,τ) and $B \in$ SGPLC (Y, σ). Then there exist sgp-open sets M and M^{l} of (X,τ) and (Y, σ) and sgp-closed sets N and N^{l} of X and Y respectively such that $A = M \cap N$ and $B = M^{l} \cap N^{l}$.

Then $A \times B = (M \times M^{l}) \cap (N \times N^{l})$ holds. Hence $A \times B \in$ SGPLC $(X \times Y, \tau \times \sigma)$.

2) Let $A \in$ SGPLC*(X,τ) and $B \in$ SGPLC*(Y, σ). Then there exist sgp-open sets K and K^{l} of (X,τ) and (Y, σ) and sgp-closed sets L and L^{l} of X and Y respectively such that $A = K \cap L$ and $B = K^{l} \cap L^{l}$.

Then $A \times B = (K \times K^{l}) \cap (L \times L^{l})$ holds. Hence $A \times B \in$ SGPLC*$(X \times Y, \tau \times \sigma)$.

3) Let $A \in$ SGPLC** (X,τ) and $B \in$ SGPLC**(Y, σ). Then there exist open sets W and W^l of (X,τ) and (Y, σ) and sgp-closed sets V and V^l of X and Y respectively such that $A = W \cap V$ and $B = W^l \cap V^l$.

Then $A \times B = (W \times W^l) \cap (V \times V^l)$ holds.

Hence $A \times B \in$ SGPLC**$(X \times Y, \tau \times \sigma)$.

Definition 1.2.10: A topological space (X,τ) is said to be sgp-submaximal if every dense subset in it is sgp-open.

Theorem 1.2.11: Every submaximal space is sgp-submaximal.

Proof: Let (X,τ) be a submaximal space and A be a dense subset of (X,τ). Then A is open. But every open set is sgp-open and so A is sgp-open. Therefore (X,τ) is sgp-submaximal.

The converse of the above theorem need not be true as seen from the following example.

Example 1.2.12: In the Example 6.2.11, the space (X,τ) is sgp-submaximal but not submaximal, every dense subset is sgp-open. However the set A= $\{a, b\}$ is dense in (X,τ), but it is not open in X. Therefore (X,τ) is not submaximal.

Theorem 1.2.13: Every ω-submaximal space is sgp-submaximal.

Proof: Let (X,τ) be a ω-submaximal space and A be a dense subset of (X,τ). Then A is ω-open. But every ω-open set is sgp-open and so A is sgp-open. Therefore (X,τ) is sgp-submaximal.

The converse of the above theorem need not be true as seen from the following example.

Example 1.2.14: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}\}$. Then the space (X,τ) is sgp-submaximal but not an ω-submaximal.

Remark 1.2.15: g-submaximals and sgp-submaximals are independent as seen from the following examples.

Example 1.2.16: In the Example 6.2.31, the space (X,τ) is g-submaximal but not a sgp-submaximal, because for the subset $\{a, c\}$ is dense in (X,τ) it is not a sgp-open set in (X,τ) but it is g-open in (X,τ).

Example 1.2.17: Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a, b\}\}$. Then the space (X,τ) is sgp-submaximal but not a g-submaximal, because for the subset $\{b, c\}$ is dense in (X,τ) it is not a g-open set in (X,τ) but it is sgp-open in (X,τ).

REFERENCES

Abd-El - Monsef, M.E., El-Deeb, S.N. and Mahmoud, R.A., β - open and β - continuous mappings, Bull. Fac. Sci. Assiut Univ. 12 (1983), 70 – 90.

D. Andrijevic, Semi-preopen sets, Mat. Vesnik 38 (1986), No.1, 24 – 32.

K. Balachandran, P. Sundaram and H. Maki, On generalized locally closed sets and GLC-continuous functions, Indian.Pure Appl. Math., 27(3),(1996),235-244.

N. Biswas , Characterization of semicontinuous mappings, Atti. Accad. Naz. Lience. Rend. Cl. Sci. Fis. Mat. Nat (8), 48 (1970), 399 – 402.

N. Bourbaki, General topology, Part I, Addison-Wesley, Reading, Mass, 1966.

M. Ganster and I. L. Reilly, Locally closed sets and LC-continuous functions, Internal J. Math. and Math. Sci., 12(1989), 417-424.

Y. Gnanambal, Studies on generalized pre-regular closed sets and generalization of locally closed sets, Ph.D, Thesis Bharathiar University, Coimbatore, (1998).

C. Kuratowski and W. Sierpinski, Sur les differences deux ensembles fermes, Tohoku Math .J., 20(1921), 22-25.

N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly 70 (1963), 36-41.

Mashhour, A.S., I.A. Hasanein and S.N. El-Deeb, On α -continuous and α - open mappings, Acta. Math Hunga. 41 (1983), 213 – 218.

Mashhour, A.S., Abd El-Monsef, M.E., and I.A. Hasanein: On pretopological spaces, Bull. Math. Soc.Sci.Math. R.S.R. 28(76) (1984), 39-45.

Govindappa Navalagi, and Girishsajjanshettar, (2012),Some properties of (s, p)-open and (s, p)-closed functions, Int. J. of Matematical sciences and applications,Vol. 2 No.1(2012),Pp. 13-16.

G. B. Navalagi, Completely preirresolute functions and completely gp-irresolute functions, IJMMS

J. H. Park and J. K. Park, On semi-generalized locally closed sets, and SGLC-continuous functions, Indian Jl. Pure Appl. Math., 31(9) (2000), 1103-1112.

M. Sheik John, A study on generalizations of closed sets and continuous maps in topological spaces and bitopological spaces, Ph.D., thesis, Bharathiar University, Coimbatore (2002).